What is next in the early Universe cosmology?

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WHAT NEXT?
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History of the Universe and outcome of recent CMB observations

Inflationary spectral predictions and observations

Remaining viable inflationary models

$f(R)$ gravity and $R + R^2$ inflationary model

Relation to BEH inflation in scalar-tensor gravity

Conclusions - what is next?

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)\,.$$

The history of the Universe in one line according to the present paradigm:

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Four fundamental cosmological constants

One-to-one relation to the four epochs of the history of the Universe. A fundamental theory outside the standard model of elementary particles – "new physics" – beyond each of these constants.

- Characteristic amplitude of primordial scalar (adiabatic) perturbations.

$$\Delta_\zeta^2 = 2.2 \times 10^{-9} \ , \quad P_s(k) = \int \frac{\Delta_\zeta^2}{k} dk$$

  Theory of initial conditions – inflation.

- Baryon to photon ratio.

$$\frac{n_b}{n_\gamma} = 6.01 \times 10^{-10} \, \frac{\Omega_b h^2}{0.0022} \left(\frac{2.725}{T_\gamma(\mathrm{K})} \right)^3$$

  Theory of baryogenesis.

- Baryon to total non-relativistic matter density.

$$\frac{\rho_b}{\rho_m} = 0.167 \, \frac{\Omega_b}{0.05} \, \frac{0.3}{\Omega_m}$$

Theory of dark matter.

- Energy density of present dark energy.

$$\rho_{DE} = \frac{\epsilon_{DE}}{c^2} = 6.44 \times 10^{-30} \, \frac{\Omega_{DE}}{0.70} \, \left(\frac{H_0}{70}\right)^2 \, \mathrm{g/cm^3}$$

$$\frac{G^2 \hbar \epsilon_{DE}}{c^7} = 1.25 \times 10^{-123} \, \frac{\Omega_{DE}}{0.70} \, \left(\frac{H_0}{70}\right)^2$$

Theory of present dark energy (of a cosmological constant).

The minimal present standard cosmological model

ΛCDM $+ (\mathcal{K} = 0)$+(scale-invariant adiabatic perturbations) contains two more parameters:

- H_0 – not a constant, but a present value of $H(t)$;
- $\tau \approx 0.09$ – optical width after recombination – a constant, but not fundamental.

4 fundamental cosmological constants $\Longrightarrow$ no more than 4 cosmological "coincidences", all other "coincidences" exist already at the level of usual laboratory physics.

Outcome of recent CMB observations

The most important for the history of the early Universe are:

1. The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N^{-1}$ has been discovered:

$$P_\zeta(k) = \int \frac{\Delta_\zeta^2(k)}{k} dk, \quad \Delta_\zeta^2 = \left(2.20^{+0.05}_{-0.06}\right) 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \mathrm{Mpc}^{-1}, \quad n_s - 1 = -0.040 \pm 0.007$$

N.B.: The value is obtained under some natural assumptions, the most critical of them is $N_\nu = 3$, for $N_\nu = 4$ many things have to be reconsidered.

Thus, the fifth fundamental cosmological number is discovered. However, the theory (some models of inflation) can derive it. Actually, it was predicted almost 30 years before the discovery.

2. Primordial gravitational waves (GW) have not been discovered in this order , at the level of $r \sim 8|n_s - 1| \approx 0.3$.

Consequence (slightly extrapolated): $|n_t| \ll 1 - n_s$, a very flat potential in the Einstein frame.

Adding the assumption that the measured part of the primordial spectrum is not a special, but a typical part (a kind of "no-scale" hypothesis), greatly restricts the space of viable inflationary models.

The crucial question for the whole inflationary scenario and for further experiments: in which order in $|n_s - 1|$ may we expect primordial GW?

Generation of scalar and tensor perturbations during inflation

A genuine quantum-gravitational effect: a particular case of the effect of particle-antiparticle creation by an external gravitational field. Requires quantization of a space-time metric. Similar to electron-positron creation by an electric field. From the diagrammatic point of view: an imaginary part of a one-loop correction to the propagator of a gravitational field from all quantum matter fields including the gravitational field itself, too.

One spatial Fourier mode $\propto e^{i\mathbf{kr}}$ is considered.

For scales of astronomical and cosmological interest, the effect occurs at the primordial de Sitter (inflationary) stage when $k \sim a(t)H(t)$ where $k \equiv |\mathbf{k}|$ (the first Hubble radius crossing).

After that, for a very long period when $k \ll aH$ until the second Hubble radius crossing (which occurs rather recently at the FLRWRD or FLRWMD stages), there exist one mode of scalar (adiabatic, density) perturbations and two modes of tensor perturbations (primordial gravitational waves) for which metric perturbations are constant (in some gauge) and independent of (unknown) local microphysics due to the causality principle.

In this regime in the coordinate representation:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\zeta(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

Classical-to-quantum transition

Quantum-to-classical transition: in fact, metric perturbations h_{lm} are quantum (operators in the Heisenberg representation) and remain quantum up to the present time. But, after omitting of a very small part, decaying with time, they become commuting and, thus, equivalent to classical (c-number) stochastic quantities with the Gaussian statistics (up to small terms quadratic in ζ, g).

Remaining quantum coherence: deterministic correlation between $\mathbf{k}$ and $-\mathbf{k}$ modes - shows itself in the appearance of acoustic oscillations (primordial oscillations in case of GW).

Spectral predictions of the one-field inflationary scenario in GR

Scalar (adiabatic) perturbations:

$$P_\zeta(k) = \frac{H_k^4}{4\pi^2\dot{\phi}^2} = \frac{GH_k^4}{\pi|\dot{H}|_k} = \frac{128\pi G^3 V_k^3}{3V_k'^2}$$

where the index k means that the quantity is taken at the moment $t = t_k$ of the Hubble radius crossing during inflation for each spatial Fourier mode $k = a(t_k)H(t_k)$. Through this relation, the number of e-folds from the end of inflation back in time $N(t)$ transforms to $N(k) = \ln\frac{k_f}{k}$ where $k_f = a(t_f)H(t_f)$, t_f denotes the end of inflation.
The spectral slope

$$n_s(k) - 1 \equiv \frac{d\ln P_\zeta(k)}{d\ln k} = \frac{1}{\kappa^2}\left(2\frac{V_k''}{V_k} - 3\left(\frac{V_k'}{V_k}\right)^2\right)$$

Tensor perturbations - primordial gravitational waves (A.A. Starobinsky, JETP Lett. 50, 844 (1979)):

$$P_g(k) = \frac{16GH_k^2}{\pi}; \quad n_g(k) \equiv \frac{d\ln P_g(k)}{d\ln k} = -\frac{1}{\kappa^2}\left(\frac{V_k'}{V_k}\right)^2$$

The consistency relation:

$$r(k) \equiv \frac{P_g}{P_\zeta} = \frac{16|\dot{H}_k|}{H_k^2} = 8|n_g(k)|$$

Tensor perturbations are always suppressed by at least the factor $\sim 8/N(k)$ compared to scalar ones. For the present Hubble scale, $N(k_H) = (50-60)$.

Potential reconstruction from scalar power spectrum

In the slow-roll approximation:

$$\frac{V^3}{V'^2} = CP_\zeta(k(t(\phi))), \quad C = const$$

Changing variables for ϕ to $N(\phi)$ and integrating, we get:

$$\frac{1}{V(N)} = -\frac{\kappa^2}{C}\int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int dN \sqrt{\frac{d\ln V}{dN}}$$

An ambiguity in the form of $V(\phi)$ because of an integration constant in the first equation. Information about $P_g(k)$ helps to remove this ambiguity.

In particular, if primordial GW are not discovered in the order $n_s - 1$:

$$r \ll 8|n_s - 1| \approx 0.3 \ ,$$

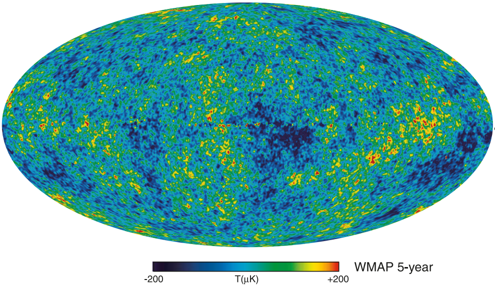
then $\left(\frac{V'}{V}\right)^2 \ll \left|\frac{V''}{V}\right|, \ |n_g| = \frac{r}{8} \ll |n_s - 1|, \ |n_g|N \ll 1$.

This is possible only if $V = V_0 + \delta V, \ |\delta V| \ll V_0$ – a plateau-like potential. Then

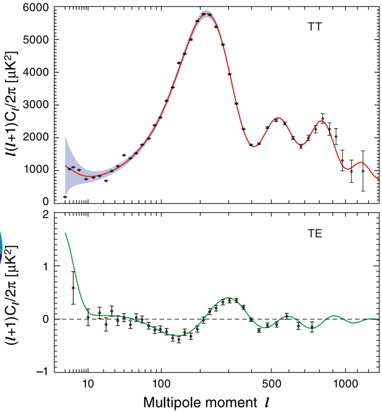
$$\delta V(N) = \frac{\kappa^2 V_0^2}{C} \int \frac{dN}{P_\zeta(N)}$$

$$\kappa\phi = \int \frac{dN}{\sqrt{V_0}} \sqrt{\frac{d(\delta V(N))}{dN}}$$

Here, integration constants renormalize V_0 and shift ϕ. Thus, the unambiguous determination of the form of $V(\phi)$ without knowledge of $P_g(k)$ becomes possible.



WMAP 5-year
-200
T(μK)
+200



TT
TE
l(l+1)C_l/2π [μK²]
(l+1)C_l/2π [μK²]
6000
5000
4000
3000
2000
1000
0
2
1
0
−1
10
100
500
1000
Multipole moment l

Combined results from Planck and other experiments

P. A. R. Ade et al., arXiv:1303.5082

Model	Parameter	*Planck*+WP	*Planck*+WP+lensing	*Planck* + WP+high-ℓ	*Planck*+WP+BAO
ΛCDM + tensor	n_s	0.9624 ± 0.0075	0.9653 ± 0.0069	0.9600 ± 0.0071	$0.9643 + 0.0059$
	$r_{0.002}$	< 0.12	< 0.13	< 0.11	< 0.12
	$-2\Delta \ln \mathcal{L}_{max}$	0	0	0	-0.31

Table 4. Constraints on the primordial perturbation parameters in the ΛCDM+r model from *Planck* combined with other data sets. The constraints are given at the pivot scale $k_* = 0.002$ Mpc^{-1}.

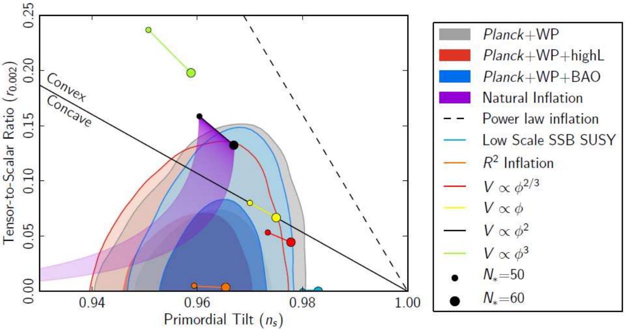


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Remaining viable models

I. Disfavoured at 95% and more CL.

1. Scale-free (or, the Harrison-Zeldovich) spectrum $n_s = 1$.
2. Power-law inflation (exponential inflaton potential $V(\phi)$).
3. Power-law $V(\phi) \propto \phi^n$ with $n \geq 2$.

II. Lying between 68% and 95% CL.

1. Other monomial potentials.
2. New inflation (or, the hill-top model with $V(\phi) = V_0 - \frac{\lambda\phi^4}{4}$).
3. Natural inflation.

Most favoured models

Models with $n_s - 1 = -\frac{2}{N} \approx -0.04$ and $r \ll 8|n_s - 1| \approx 0.32$.

1. $R + R^2$ model (AS, 1980).

2. A scalar field model with $V(\phi) = \frac{\lambda\phi^4}{4}$ at large ϕ and strong non-minimal coupling to gravity $\xi R\phi^2$ with $\xi < 0, \ |\xi| \gg 1$, including the Brout-Englert-Higgs (BEH) inflationary model.

Both these models have $r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.005$.

3. Minimally coupled GR models with a very flat inflaton potential $V(\phi)$.

Generic family of most favoured models with a scale-free power spectrum in GR

Assumptions: $n_s - 1 = -\frac{2}{N} \approx -0.04$ and $r \ll 8|n_s - 1|$ for all $N = 1 - 60$.

Consequences:
$P_\zeta(N) \propto N^2,\ |n_t| \ll |n_s - 1|,\ |n_t|N \ll 1,\ H \approx const$ during inflation.

Two parametric family:

$$V(\phi) = V_0\,(1 - \exp(-\alpha\kappa\phi))$$

with $\alpha\kappa\phi \gg 1$ but α not very small. For this family:

$$r = \frac{8}{\alpha^2 N^2}$$

$f(R)$ gravity

The simplest model of modified gravity (= geometrical dark energy) considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime.

$$S = \frac{1}{16\pi G}\int f(R)\sqrt{-g}\, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R_\mu^\mu \ .$$

One-loop corrections depending on R only (not on its derivatives) are assumed to be included into $f(R)$. The normalization point: at laboratory values of R where the scalaron mass (see below) $m_s \approx \mathrm{const}$.

Field equations

$$\frac{1}{8\pi G}\left(R^\nu_\mu - \frac{1}{2}\,\delta^\nu_\mu R\right) = -\left(T^\nu_{\mu\,(vis)} + T^\nu_{\mu\,(DM)} + T^\nu_{\mu\,(DE)}\right) \ ,$$

where $G = G_0 = const$ is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

$$8\pi G T^\nu_{\mu\,(DE)} = F'(R)\,R^\nu_\mu - \frac{1}{2}\,F(R)\delta^\nu_\mu + \left(\nabla_\mu\nabla^\nu - \delta^\nu_\mu\nabla_\gamma\nabla^\gamma\right)F'(R)\,.$$

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots $R = R_{dS}$ of the algebraic equation

$$Rf'(R) = 2f(R)\ .$$

In the special case $f(R) \propto R^2$, the de Sitter space-time with *any* curvature is a solution.

Degrees of freedom

I. In quantum language: particle content.
1. Graviton – spin 2, massless, transverse traceless.
2. Scalaron – spin 0, massive, mass - R-dependent:
$m_s^2(R) = \frac{1}{3f''(R)}$ in the WKB-regime.

II. Equivalently, in classical language: number of free functions of spatial coordinates at an initial Cauchy hypersurface.
Six, instead of four for GR – two additional functions describe massive scalar waves.

Thus, $f(R)$ gravity is a non-perturbative generalization of GR. It is equivalent to scalar-tensor gravity with $\omega_{BD} = 0$ (if $f''(R) \neq 0$).

Background FLRW equations in $f(R)$ gravity

$$ds^2 = dt^2 - a^2(t)\left(dx^2 + dy^2 + dz^2\right)$$

$$H \equiv \frac{\dot{a}}{a}\,, \quad R = 6(\dot{H} + 2H^2)$$

The trace equation (4th order)

$$\frac{3}{a^3}\frac{d}{dt}\left(a^3\frac{df'(R)}{dt}\right) - Rf'(R) + 2f(R) = 8\pi G(\rho_m - 3p_m)$$

The 0-0 equation (3d order)

$$3H\frac{df'(R)}{dt} - 3(\dot{H} + H^2)f'(R) + \frac{f(R)}{2} = 8\pi G\rho_m$$

Reduction to the first order equation

In the absence of spatial curvature and $\rho_m = 0$, it is always possible to reduce these equations to a first order one using the transformation to the Einstein frame and the Hamilton-Jacobi-like equation for a minimally coupled scalar field in a spatially flat FLRW metric:

$$\frac{2}{3\kappa^2}\left(\frac{dH_E(\phi)}{d\phi}\right)^2 = H_E^2 - \frac{\kappa^2}{3}V(\phi)$$

where

$$H_E \equiv \frac{d}{dt_E}\ln a_E = \frac{1}{\sqrt{f'}}\left(\ln a + \frac{1}{2}\ln f'\right)$$

$$= \frac{1}{2\sqrt{f'}}\left(3H + \frac{\dot{H}}{H} - \frac{f}{6Hf'}\right)$$

From a solution $H_E(\phi(R))$ of this equation, the scale factor $a(t)$ follows in the parametric form:

$$\ln a = -\frac{1}{2}\ln f'(R) - \frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 H_E(R) \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

$$t = -\frac{3}{4}\int \left(\frac{f''}{f'}\right)^2 \left(\frac{dH_E(R)}{dR}\right)^{-1} dR$$

Thus, this model is no more complicated than GR + scalar field with a potential!

Most favoured inflationary models in $f(R)$ gravity

The simplest one (Starobinsky, 1980):

$$f(R) = R + \frac{R^2}{6M^2}$$

with small one-loop quantum gravitational corrections producing the scalaron decay via the effect of particle-antiparticle creation by gravitational field (so all present matter is created in this way).

During inflation ($H \gg M$): $H = \frac{M^2}{6}(t_f - t)$, $|\dot{H}| \ll H^2$.

The only parameter M is fixed by observations – by the primordial amplitude of adiabatic (density) perturbations in the gravitationally clustered matter component:

$$M = 2.9 \times 10^{-6} M_{Pl} \, (50/N) \, ,$$

where $N \sim (50 - 55)$, $M_{Pl} = \sqrt{G} \approx 10^{19}$ GeV.

A particular (but very specific!) case of the (formally) renormalizable fourth order gravity

$$\mathcal{L} = \frac{R}{16\pi G} + AR^2 + BC_{\mu\mu\sigma\tau}C^{\mu\mu\sigma\tau}$$

$$A = \frac{N^2}{288\pi^2\Delta_\zeta^2} \approx 4.0 \times 10^8 \left(\frac{N}{50}\right)^2 \gg B$$

.

Predictions for primordial perturbation spectra:

$$n_s = 1 - \frac{2}{N} \approx 0.96$$

$$r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.005$$

.

Generic $f(R)$ inflationary model with
$n_s(k) = 1 - \frac{2}{N(k)}$; $r(k) \sim \frac{10}{N^2(k)} \ll 1 - n_s$ has
$V(\phi) = V_0\,(1 - \exp(-\alpha\kappa\phi))$ in the Einstein frame.
In the Jordan (physical) frame, this means that

$$f(R) = \frac{R^2}{6M^2} + CR^{2-\alpha\sqrt{3/2}}$$

for large R.

Less natural, has one more free parameter, cannot be used after inflation, but still possible. The additional (aesthetic) assumption that a $f(R)$ model should describe not only inflation but the whole post-inflationary evolution including the GR regime returns us back to the $R + R^2$ model with the preferred value of $\alpha = \sqrt{2/3}$.

Post-inflationary evolution

First order equation:

$$x = H^{3/2}, \;\; y = \frac{1}{2} H^{-1/2} \dot{H}, \;\; dt = \frac{dx}{3x^{2/3} y}$$

$$\frac{dy}{dx} = -\frac{M^2}{12 x^{1/3} y} - 1$$

The y-axis corresponds to inflection points $\dot{a} = \ddot{a} = 0, \dddot{a} \neq 0$.
A curve reaching the y-axis at the point $(0, y_0 < 0)$ continues from the point $(0, -y_0)$ to the right.
Late-time asymptotic:

$$a(t) \propto t^{2/3} \left(1 + \frac{2}{3Mt} \sin M(t - t_1) \right), \;\; R \approx -\frac{8M}{3t} \sin M(t - t_1)$$

$$< R^2 > = \frac{32M^2}{9t^2}, \quad 8\pi G \rho_{s,eff} = \frac{3 < R^2 >}{8M^2} = \frac{4}{3t^2} \propto a^{-3}$$

Scalaron decay and creation of matter

Transition to the FLRWRD stage: occurs through the same mechanism which has been used for generation of perturbations: creation of particle-antiparticle pairs of all quantum matter fields by fast oscillations of R. Technically: one-loop quantum corrections from all matter quantum fields have to be added to the action of the $R + R^2$ gravity. In the particle interpretation: scalaron decays into particles and particles with the energy $E = M/2$.

The most effective decay channel: into minimally coupled scalars with $m \ll M$. Then the formula obtained in Ya. B. Zeldovich and A. A. Starobinsky, JETP Lett. 26, 252 (1977) can be used:

$$\frac{1}{\sqrt{-g}}\frac{d}{dt}(\sqrt{-g}\,n_s) = \frac{R^2}{576\pi}$$

The corresponding (partial) decay rate is $\Gamma = \frac{GM^3}{24} \sim 10^{24}\ \mathrm{s}^{-1}$, that leads to the maximal temperature $T \approx 3 \times 10^9$ GeV at the beginning of the FLRWRD stage and to $N \approx 53$ for the reference scale in the CMB measurements ($k/a(t_0) = 0.05\ \mathrm{Mpc}^{-1}$), see D. S. Gorbunov, A. G. Panin, Phys. Lett. B 700, 157 (2011) and F. Bezrukov, D. Gorbunov, Phys. Lett. B 713, 365 (2012) for more details.

One viable microphysical model leading to such form of $f(R)$

A non-minimally coupled scalar field with a large negative coupling ξ (for this choice of signs, $\xi_{conf} = \frac{1}{6}$):

$$L = \frac{R}{16\pi G} - \frac{\xi R\phi^2}{2} + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi), \quad \xi < 0, \quad |\xi| \gg 1 \, .$$

Leads to $f' > 1$.
Recent development: the BEH inflationary model (F. Bezrukov and M. Shaposhnikov, 2008). In the limit $|\xi| \gg 1$, the BEH scalar tree level potential $V(\phi) = \frac{\lambda(\phi^2 - \phi_0^2)^2}{4}$ just produces $f(R) = \frac{1}{16\pi G}\left(R + \frac{R^2}{6M^2}\right)$ with $M^2 = \lambda/24\pi\xi^2 G$ and $\phi^2 = |\xi|R/\lambda$ (for this model, $|\xi|G\phi_0^2 \ll 1$).

SM loop corrections to the tree potential leads to $\lambda = \lambda(\phi)$, then the same expression for $f(R)$ follows with

$$M^2 = \frac{\lambda(\phi(R))}{24\pi\xi^2 G}\left(1 + \mathcal{O}\left(\frac{d\ln\lambda(\phi(R))}{d\ln\phi}\right)^2\right).$$

.

The approximate shift invariance $\phi \to \phi + c,\ \ c = \mathrm{const}$ permitting slow-roll inflation for a minimally coupled inflaton scalar field transforms here to the approximate scale (dilatation) invariance

$\phi \to c\phi,\ \ R \to c^2 R,\ \ x^\mu \to x^\mu/c,\ \ \mu = 0, ..3$

for curvatures exceeding that at the end of inflation in the physical (Jordan) frame. Of course, this symmetry needs not be fundamental, i.e. existing in some more microscopic model at the level of its action.

Conclusions

- The fifth fundamental cosmological number has been discovered, but the theory has been ready to derive and predict it.
- There exists a class of inflationary models having $n_s - 1 \approx -0.04$ and $r \ll 8|n_s - 1| \approx 0.32$ which is most favoured by the Planck and other observational data.
- This class includes the one-parametric pioneer $R + R^2$ and BEH inflationary models in modified (scalar-tensor) gravity, and more general two-parametric models including a GR model with a very flat inflaton potential.
- Natural extrapolation of existing data, namely the hypothesis that the observed interval of N is not special, leads to the expectation of a small, but not too small tensor-to-scalar ratio $r \sim 10/N^2 \sim 3(n_s - 1)^2$. The preferred value in the most elegant models with one free parameter is $r = 12/N^2 = 3(n_s - 1)^2 \approx 5 \times 10^{-3}$.

- This value of r is well possible to measure in future. In particular, the PRISM (Polarized Radiation Imaging and Spectroscopy Mission – see the description in arXiv:1310.1554) planned to reach $r \sim 3 \times 10^{-4}$ at 3σ as a large-class ESA mission. May be reduced to a middle-class mission since $r \sim 10^{-3}$ seems to be a sufficient accuracy.

  What can be achieved in the case of positive detection:

  1. Discovery of primordial gravitational waves.
  2. Decisive test of a narrow class of inflationary models.
  3. Decisive test of the inflationary paradigm as a whole.
  4. Decisive argument for the necessity of quantization of gravitational waves.